



Fuzzy Semi- G – Regular And Fuzzy Semi – G – Normal Spaces

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Abstract

Objective: The aim of this paper is to introduce and study fuzzy sets and fuzzy point or the properties fuzzy sets and study two new classes of spaces, called Fuzzy semi-g-regular and fuzzy semi-g-normal spaces. Method: Fuzzy Semi-g-regularity and fuzzy semi-g-normality are separation properties obtained by utilizing fuzzy semi-generalized closed sets. Results: Recall that a fuzzy topological space (X, τ) is called fuzzy semi-generalized closed, briefly f s g-closed, if the fuzzy semi-closure of $\tilde{A} \leq X$ is a fuzzy of $U \leq X$ whenever \tilde{A} is a fuzzy set of U and U is fuzzy semi-open in (X, τ) . Novelty: two new classes of spaces, called Fuzzy semi-g-regular and fuzzy semi-g-normal spaces.

Keywords

fuzzy sets, fuzzy point, fuzzy topological spaces (X, τ) , fuzzy generalized closed (f s g - closed), fuzzy semi -regular, fuzzy semi normal, fuzzy semi g-regular, fuzzy semi g – normal spaces.

Introduction

In 1970, Levine [1] presented a new and important view in General Topology, namely the view of a fuzzy generalized closed set. A fuzzy topological space (X, τ) is called fuzzy generalized closed, briefly f g-closed, if $cl(\tilde{A}) \leq \tilde{U}$ whenever $\tilde{A} \leq \tilde{U}$ and is \tilde{U} open in (X, τ) . This view has been studied extensively in recent years by many fuzzy topologists. The investigation of fuzzy generalized closed sets has led to several new and interesting concepts. In 1987 [2], a fuzzy generalized the usual notions of fuzzy regularity and fuzzy normality by replacing "fuzzy closed set" with "fuzzy g-closed set" in the definitions, thus obtaining the notions of f g-regularity and f g-normality.

The aim of our paper is to introduce and investigate the views of fuzzy semi-g-regularity

and fuzzy semi-g-normality. We shall do so by utilizing the concept of a fuzzy semi-generalized closed set [3].

a fuzzy topological space (\tilde{X}, \tilde{T}) is called fuzzy semi-open if $\tilde{A} \leq \text{cl}(\tilde{A}^\circ)$, where $\text{cl}(\tilde{A})$ and $(\tilde{A})^\circ$ denote the closure and the interior of \tilde{A} . The complement of a fuzzy semi-open set is called a fuzzy semi-closed set [4]. The fuzzy semi-closure of \tilde{A} , denoted by $f(\text{cl}(\tilde{A}^\circ))$ is the intersection of all fuzzy semi-closed sets containing \tilde{A} . The fuzzy semi-interior of \tilde{A} , denoted by $f(\tilde{A}^\circ)$ is the largest fuzzy semi-open set contained in $(\tilde{A})^\circ$. It is well known that $f(\text{cl}(\tilde{A}^\circ)) = \tilde{A} \cap \text{cl}(\tilde{A}^\circ)$ and $f(\tilde{A}^\circ) = \tilde{A} \cap (f(\tilde{A}^\circ))$ for any fuzzy set $\tilde{A} \leq \tilde{X}$. Recall that a fuzzy set \tilde{A} is said to be fuzzy preopen if $\tilde{A} \leq f(\tilde{A}^\circ)$.

Basic concepts of fuzzy set .

Definition 1 : [5] Let X be a nonempty set , a fuzzy set \tilde{A} in X is characteristic by a membership function , $\mu_{\tilde{A}} : X \rightarrow [0,1]$. and we can write this fuzzy set as,

$$\tilde{A} = \{ (X, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \leq 1 \}$$

The collection of all fuzzy sets in X will be denoted by I^X , where $I^X = \{ \tilde{A} : \tilde{A} \text{ is fuzzy set in } X \}$.

Definition 2 : [1] The support of a fuzzy set \tilde{A} is the set of all $x \in X$ such that $\mu_{\tilde{A}}(x) > 0$,and is denoted by $S(\tilde{A})$.

Definition 3 : [6] A fuzzy point \tilde{P}^r_x in X is a special fuzzy set with membership function defined by.

$$\tilde{P}^r_x = \begin{cases} r, & \text{if } x = y \\ 0, & \text{if } x \neq y \end{cases}, \text{ where } 0 < r \leq 1, y \text{ is the support of } \tilde{P}^r_x(x).$$

Definition 4 : [7] A fuzzy set \tilde{A} is said to be finite fuzzy set if $S(\tilde{A})$ is a finite set.

Remark 5 :

1) A nonempty set X is a fuzzy set with membership $\mu_x(X) = 1, \forall x \in X$

And X is called a crisp set .

2) A membership function $\mu_{\tilde{\emptyset}}(x) = 0, \forall \emptyset \in X$. is called an empty set and denoted by $\tilde{\emptyset}$.

Definition 6 : [8] Let \tilde{P}^r_x be a fuzzy point and \tilde{C} be a fuzzy set is a nonempty set \tilde{X} then \tilde{P}^r_x is said to be in \tilde{C} or \tilde{C} contains \tilde{P}^r_x if $\mu_{\tilde{P}^r_x} \leq \mu_{\tilde{C}}(x), \forall x \in X$ and denoted by $X \in S(\tilde{C})$ [9], [10], [11].

Definition 7 : [9] Let \tilde{A} and \tilde{B} be a fuzzy sets of a universal set X then,

- 1) $\tilde{A} \leq \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x), \forall x \in X$.
- 2) $\tilde{A} = \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x), \forall x \in X$.
- 3) \tilde{A}^c is the complement of a fuzzy set \tilde{A} with membership function $\mu_{\tilde{A}^c} = 1 - \mu_{\tilde{A}}(x)$ [12].
- 4) $\tilde{C} = \tilde{A} \cup \tilde{B}$ if and only if $\mu_{\tilde{C}}(x) = \text{Max} \{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}, \forall x \in X$.
- 5) $\tilde{D} = \tilde{A} \cap \tilde{B}$ if and only if $\mu_{\tilde{D}}(x) = \text{Min} \{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}, \forall x \in X$.
- 6) More generally, for a family of fuzzy sets $\{ \tilde{A}_\alpha : \alpha \in \Lambda \}$ where Λ is the any index set } the union $\tilde{C} = \cup \tilde{A}_\alpha$.

And the intersection $\tilde{D} = \cap \tilde{A}_\alpha$, and defined respectively by.

$$\mu_{\tilde{C}}(x) = \sup \{ \mu_{\tilde{A}_\alpha}(x), x \in X \}, \mu_{\tilde{D}}(x) = \inf \{ \mu_{\tilde{A}_\alpha}(x) : x \in X \}.$$

Methods

preliminaries

Definition 1 : A fuzzy topological space (\tilde{X}, \tilde{T}) is said to be

1. fuzzy semi-regular [10] if for each fuzzy semi-closed a set $\tilde{A} \leq \tilde{X}$ and each fuzzy point $P_x \geq \tilde{A}$ here exist disjoint fuzzy semi-open sets $\tilde{U}, \tilde{V} \leq \tilde{X}$ such that $P_x \geq \tilde{U}$ and $\tilde{A} \leq \tilde{V}$,
2. fuzzy semi-normal if for each pair $\tilde{A}, \tilde{B} \leq \tilde{X}$ of disjoint fuzzy semi-closed sets there exist disjoint fuzzy semi-open sets $\tilde{U}, \tilde{V} \leq \tilde{X}$ such that $\tilde{A} \leq \tilde{U}$ and $\tilde{B} \leq \tilde{V}$.

Definition 2 : A fuzzy topological space (\tilde{X}, \tilde{T}) is called fuzzy semi-generalized closed, briefly f s g-closed, if $f s(\text{cl}(\tilde{A})) \leq \tilde{U}$ whenever $\tilde{A} \leq \tilde{U}$ and \tilde{U} is fuzzy semi-open.

The complement of a fuzzy semi-generalized closed set is called fuzzy semi-generalized open.

It is obvious that every fuzzy semi-closed set is f s g-closed. Spaces in which every f s g-closed set

is fuzzy semi-closed have been called fuzzy semi- $\tilde{T}_{1/2}$. One can easily show that a space (\tilde{X}, \tilde{T}) semi- $\tilde{T}_{1/2}$ if and only if every singleton is either fuzzy semi-open or fuzzy semi-closed. Recall also that a space (\tilde{X}, \tilde{T}) is said to be fuzzy semi- \tilde{T}_2 if distinct fuzzy points can be separated by disjoint fuzzy semi-open set.

Results and Discussions

In, Jankovic and Reilly pointed out that every singleton $\{\tilde{x}\}$ of a space (\tilde{X}, \tilde{T}) is either nowhere fuzzy dense or fuzzy preopen. This yields a decomposition $\tilde{X} = \tilde{X}_1 \cup \tilde{X}_2$, where $\tilde{X}_1 = \{P_x \leq \tilde{X} : \{\tilde{x}\} \text{ is nowhere fuzzy dense}\}$ and $\tilde{X}_2 = \{P_x \leq \tilde{X} : \{\tilde{x}\} \text{ is fuzzy preopen}\}$ [13]. The usefulness of this decomposition, which we call the Jankovic-Reilly decomposition, is illustrated by the following result.

Lemma 2.3 : A fuzzy space (\tilde{X}, \tilde{T}) is f s g-closed if and only if $\tilde{X}_1 \cap f s(\text{cl}(\tilde{A})) \leq \tilde{A}$.

Definition 4 : A fuzzy function $\tilde{f} : (\tilde{X}, \tilde{T}) \rightarrow (\tilde{Y}, \tilde{\sigma})$ is called

- (i) fuzzy irresolute if $\tilde{f}^{-1}(\tilde{B})$ is fuzzy semi-open in \tilde{X} for every fuzzy semi-open set $\tilde{B} \leq \tilde{Y}$,
- (ii) fuzzy pre-semi-closed if $\tilde{f}(\tilde{F})$ is fuzzy semi-closed in \tilde{Y} for every fuzzy semi-closed set $\tilde{F} \leq \tilde{X}$

Lemma 2.5 : A fuzzy function $\tilde{f} : (\tilde{X}, \tilde{T}) \rightarrow (\tilde{Y}, \tilde{\sigma})$ is fuzzy pre-semi-closed if and only if $f s(\text{cl}(\tilde{f}(\tilde{A}))) \leq \tilde{f}(f s(\text{cl}(\tilde{A})))$ for every fuzzy set $\tilde{A} \leq \tilde{X}$

fuzzy Semi-g-regular spaces

Definition 6 : A fuzzy topological space (\tilde{X}, \tilde{T}) is said to be fuzzy semi-g-regular if for each f s g-closed set \tilde{A} and each fuzzy point $P_x \geq \tilde{A}$, there exist disjoint fuzzy semi-open sets $\tilde{U}, \tilde{V} \leq \tilde{X}$ such that $\tilde{A} \leq \tilde{U}$ and $P_x \leq \tilde{V}$.

Lemma 2.7:- A fuzzy space (\tilde{X}, \tilde{T}) is fuzzy semi-g-regular if and only if (\tilde{X}, \tilde{T}) is fuzzy semi-regular and fuzzy semi- $\tilde{T}_{1/2}$

Proof. : Suppose that (\tilde{X}, \tilde{T}) is fuzzy semi-g-regular. Then clearly (\tilde{X}, \tilde{T}) is fuzzy semi-regular. Now let $\tilde{A} \leq \tilde{X}$ be f s g-closed. For each $P_x \geq \tilde{A}$ there exists a fuzzy semi-open set \tilde{V}_x containing P_x such that $\tilde{V}_x \cap \tilde{A} = \tilde{\emptyset}$. If $\tilde{V} = \cup\{\tilde{V}_x : P_x \geq \tilde{A}\}$, then \tilde{V} is fuzzy semi-open and $\tilde{V} = \tilde{X} \setminus \tilde{A}$, hence \tilde{A} is fuzzy semi-closed.

The converse is obvious.

We will now show that there exist fuzzy semi-regular spaces which are not fuzzy semi-g-regular.

Recall that a space (\tilde{X}, \tilde{T}) is said to be fuzzy locally indiscrete if every fuzzy open is fuzzy closed. It is well known that (\tilde{X}, \tilde{T}) is fuzzy locally indiscrete if and only if every fuzzy set of \tilde{X} is fuzzy preopen. If (\tilde{X}, \tilde{T}) is fuzzy locally indiscrete, then every fuzzy semi-closed set is fuzzy closed and thus fuzzy clopen, and, by Lemma 1.1, every fuzzy set of \tilde{X} is f s g-closed.

Example 2.8 :- Let \tilde{Y} and \tilde{Z} be disjoint fuzzy infinite sets, and let $\tilde{X} = \tilde{Y} \vee \tilde{Z}$. Then $\tilde{T} = \{\tilde{\emptyset}, \tilde{Y}, \tilde{Z}, \tilde{X}\}$ is a fuzzy topology on \tilde{X} . Clearly (\tilde{X}, \tilde{T}) is fuzzy locally indiscrete and thus every fuzzy semi-closed set is fuzzy clopen, hence (\tilde{X}, \tilde{T}) is fuzzy semi-regular. If $\tilde{\emptyset} \neq \tilde{A} \leq \tilde{Y}$ and $P_x \geq \tilde{Y} \setminus \tilde{A}$ then \tilde{A} is f s g-closed, but \tilde{A} and P_x cannot be separated by disjoint fuzzy semi-open sets. Thus (\tilde{X}, \tilde{T}) fails to be fuzzy semi-g-regular.

Our next result characterizes fuzzy semi-regular spaces. We shall call fuzzy set $\tilde{A} \leq \tilde{X}$ fuzzy semi-clopen in (\tilde{X}, \tilde{T}) if \tilde{A} is both fuzzy semi-open and fuzzy semi-closed. Observe that for any fuzzy semi-open set \tilde{V} , f s (cl(\tilde{V})) is always fuzzy semi-clopen.

Theorem 2.9 : For a fuzzy topological space (\tilde{X}, \tilde{T}) the following are equivalent:

1. (\tilde{X}, \tilde{T}) is fuzzy semi-g-regular.
2. Every f s g-open set \tilde{U} is a union of fuzzy semi-clopen sets.
3. Every f s g-closed set \tilde{A} is an intersection of fuzzy semi-clopen sets.

Proof. (1)

$\Rightarrow(2)$: Let \tilde{U} be f s g-open and let $P_x \leq \tilde{U}$. If $\tilde{A} = \tilde{X} \setminus \tilde{U}$, then \tilde{A} is f s g-closed. By assumption there exist disjoint fuzzy semi-open set \tilde{W}_1 and \tilde{W}_2 of \tilde{X} such that $P_x \leq \tilde{W}_1$ and $\tilde{A} \leq \tilde{W}_2$. If $\tilde{V} = \text{f s}(\text{cl}(\tilde{W}_1))$, then \tilde{V} is fuzzy semi-clopen and $\tilde{V} \cap \tilde{A} \leq \tilde{V} \cap \tilde{W}_2 = \tilde{\emptyset}$. It follows that $P_x \leq \tilde{V} \leq \tilde{U}$. Thus \tilde{U} is a union of fuzzy semi-clopen sets.

(2) (3): This is obvious.

(3) \Rightarrow (1) : Let \tilde{A} be f s g-closed and let $P_x \geq \tilde{A}$. By assumption, there exists a fuzzy semi-clopen set \tilde{V} such that $\tilde{A} \leq \tilde{V}$ and $P_x \geq \tilde{V}$. If $\tilde{U} = \tilde{X} \setminus \tilde{V}$, then \tilde{U} is a fuzzy semi-open set containing P_x and $\tilde{U} \wedge \tilde{V} = \tilde{\emptyset}$. Thus (\tilde{X}, \tilde{T}) is fuzzy semi-g-regular.

F s g-open sets give rise to various separation properties of which we offer the following.

Definition 10 :- A fuzzy topological space (\tilde{X}, \tilde{T}) is called a

1. f s g- \tilde{T} space if for each pair of distinct fuzzy points there exists an f s g-open set containing one fuzzy point but not the other.
2. (f s, f s g)- \tilde{R} space if f s (cl($\{\tilde{x}\}$)) $\leq \tilde{U}$ whenever \tilde{U} is f s g-open and $P_x \leq \tilde{U}$.

Observe that the fuzzy space in Example 2.2 is neither f s g- \tilde{T} nor (f s, f s g)- \tilde{R} . Also note that every fuzzy semi- \tilde{T}_2 space is fuzzy semi- $\tilde{T}_{1/2}$, and every fuzzy semi- $\tilde{T}_{1/2}$ space is f s g- \tilde{T} .

Theorem 2.11 : Every fuzzy semi-g-regular space (\tilde{X}, \tilde{T}) is both fuzzy semi- \tilde{T}_2 and (f s, f s g)- \tilde{R} .

Proof. : Let (\tilde{X}, \tilde{T}) be fuzzy semi-g-regular, and let $P_x, P_y \leq \tilde{X}$ such that $P_x \neq P_y$. By Lemma 2.1 $\{\tilde{x}\}$ is either fuzzy semi-open or fuzzy semi-closed. If $\{\tilde{x}\}$ is fuzzy semi-open, hence f s g-open, then $\{\tilde{x}\}$ is fuzzy semi-clopen by Theorem 2.3. Thus $\{\tilde{x}\}$ and $\tilde{X} \setminus \{\tilde{x}\}$ are separating fuzzy semi-open sets. If $\{\tilde{x}\}$ is fuzzy semi-closed, then $\tilde{X} \setminus \{\tilde{x}\}$ is fuzzy semi-open and so, by Theorem 2.3, the union of fuzzy semi-clopen sets. Hence there is a fuzzy semi-clopen set $\tilde{V} \leq \tilde{X} \setminus \{\tilde{x}\}$ containing \tilde{y} . This proves that (\tilde{X}, \tilde{T}) is fuzzy semi- \tilde{T}_2 .

By Theorem 2.3 it follows immediately that (\tilde{X}, \tilde{T}) also has to be (f s, f s g)- \tilde{R} .

fuzzy Semi-g-normal spaces

Definition 1 : A fuzzy topological space (\tilde{X}, \tilde{T}) is said to be fuzzy semi-g-normal if for every

pair of disjoint f s g-closed sets \tilde{A} and \tilde{B} of (\tilde{X}, \tilde{T}) , there exist disjoint fuzzy semi-open sets $\tilde{U}, \tilde{V} \leq \tilde{X}$ such that $\tilde{A} \leq \tilde{U}$ and $\tilde{B} \leq \tilde{V}$.

Remark 3.2 Obviously every fuzzy semi-g-normal space is fuzzy semi-normal. Example 2.2 shows that the converse is false.

Recall that a fuzzy space (\tilde{X}, \tilde{T}) is said to be fuzzy semi-symmetric if for any two fuzzy points $P_x, P_y \leq \tilde{X}$, if $P_x \leq f s (cl(\{y\}))$ then $P_y \leq f s (cl(\{x\}))$. Caldas has shown that a fuzzy space (\tilde{X}, \tilde{T}) is fuzzy semi-symmetric if and only if $\{x\}$ is f s g-closed for every $P_x \leq \tilde{X}$.

Remark 3.3 : Observe that, if (\tilde{X}, \tilde{T}) is fuzzy semi-normal and $\tilde{F} \wedge \tilde{A} = \tilde{\emptyset}$, where \tilde{F} is fuzzy semi-closed and \tilde{A} is f s g-closed, then $\tilde{F} \wedge f s (cl(\tilde{A})) = \tilde{\emptyset}$, hence there exist disjoint fuzzy semi-open sets $\tilde{U}, \tilde{V} \leq \tilde{X}$ such that $\tilde{F} \leq \tilde{U}$ and $\tilde{A} \leq \tilde{V}$.

As a consequence we have the following result.

Theorem 3.4 :

1. Every fuzzy semi-normal, fuzzy semi-symmetric topological space (\tilde{X}, \tilde{T}) is fuzzy semi-regular.
2. Every fuzzy semi-g-normal, fuzzy semi-symmetric topological space (\tilde{X}, \tilde{T}) is fuzzy semi-g-regular [14], [15].

Proof. (i) Suppose that $\tilde{A} \leq \tilde{X}$ is fuzzy semi-closed and $P_x \geq \tilde{A}$. Then $\{x\}$ is f s g-closed. By Remark 3.2, \tilde{A} and $\{x\}$ can be separated by disjoint fuzzy semi-open sets.

(ii) is obvious.

Theorem 3.5 : For a fuzzy topological space (\tilde{X}, \tilde{T}) the following are equivalent:

1. (\tilde{X}, \tilde{T}) is fuzzy semi-g-normal.
2. For every f s g-closed set \tilde{A} and every f s g-open set \tilde{U} containing \tilde{A} , there is a fuzzy semi-clopen set \tilde{V} such that $\tilde{A} \leq \tilde{V} \leq \tilde{U}$.

Proof.: (1) \Rightarrow (2) : Let \tilde{A} be f s g-closed and \tilde{U} be f s g-open with $\tilde{A} \leq \tilde{U}$. Now we have $An(\tilde{X} \setminus \tilde{U}) = \tilde{\emptyset}$, hence there exist disjoint fuzzy semi-open sets \tilde{W}_1 and \tilde{W}_2 such that $\tilde{A} \leq \tilde{W}_1$ and $\tilde{X} \setminus \tilde{U} \leq \tilde{W}_2$. If $\tilde{V} = f s (cl(\tilde{W}_1))$, then \tilde{V} is fuzzy semi-clopen satisfying $\tilde{A} \leq \tilde{V} \leq \tilde{U}$. (2) (1): This is obvious.

We now investigate the behavior of fuzzy semi-g-normal spaces under certain mappings.

Lemma 3.6 : Let $\tilde{f} : (\tilde{X}, \tilde{T}) \rightarrow (\tilde{Y}, \tilde{\sigma})$ be fuzzy irresolute and fuzzy pre-semi-closed. If $\tilde{B} \leq \tilde{Y}$ is f s g-closed, then $\tilde{f}^{-1}(\tilde{B}) \leq \tilde{X}$ is f s g-closed.

Proof. Let $\tilde{B} \leq \tilde{Y}$ be f s g-closed, and let $\tilde{f}^{-1}(\tilde{B}) \leq \tilde{U}$ where $\tilde{U} \leq \tilde{X}$ is fuzzy semi-open. If $\tilde{V} = \tilde{Y} \setminus \tilde{f}(\tilde{X} \setminus \tilde{U})$, then $\tilde{V} \leq \tilde{Y}$ is fuzzy semi-open since \tilde{f} is fuzzy pre-semi-closed. Moreover, we have $\tilde{f}^{-1}(\tilde{V}) \leq \tilde{U}$ and $\tilde{B} \leq \tilde{V}$. Since \tilde{B} is f s g-closed, we conclude that $f s (cl(\tilde{B})) \leq \tilde{V}$. Since \tilde{f} is fuzzy irresolute, we now have $f s (cl \tilde{f}^{-1}(\tilde{B})) \leq \tilde{f}^{-1}((scl(\tilde{B}))^\sim) \leq \tilde{f}^{-1}(\tilde{V}) \leq \tilde{U}$. Hence $\tilde{f}^{-1}(\tilde{B})$ is f s g-closed.

Theorem 3.7 : Let $\tilde{f} : (\tilde{X}, \tilde{T}) \rightarrow (\tilde{Y}, \tilde{\sigma})$ be fuzzy irresolute, fuzzy pre-semi-closed and onto. If (\tilde{X}, \tilde{T}) is fuzzy semi-g-normal, then $(\tilde{Y}, \tilde{\sigma})$ is fuzzy semi-g-normal.

Conclusion

Proof. : Let $\tilde{B}_1, \tilde{B}_2 \leq \tilde{Y}$ be disjoint f s g-closed sets. By Lemma 3.5, $\tilde{f}^{-1}(\tilde{B}_1)$ and $\tilde{f}^{-1}(\tilde{B}_2)$ are f s g-closed, hence there exist disjoint fuzzy semi-open sets $\tilde{U}_1, \tilde{U}_2 \leq \tilde{X}$ such that $\tilde{f}^{-1}(\tilde{B}_1) \leq \tilde{U}_1$ and $\tilde{f}^{-1}(\tilde{B}_2) \leq \tilde{U}_2$. As in the proof of Lemma 3.5 there exist fuzzy semi open sets $\tilde{V}_1, \tilde{V}_2 \leq \tilde{Y}$ such that $\tilde{B} \leq \tilde{V}_i$ and $\tilde{f}^{-1}(\tilde{V}_i) \leq \tilde{U}_i$ for $i = 1, 2$. Hence $\tilde{f}^{-1}(\tilde{V}_1) \wedge \tilde{f}^{-1}(\tilde{V}_2) = \tilde{\emptyset}$ and, since \tilde{f} is onto, we have $\tilde{V}_1 \wedge \tilde{V}_2 = \tilde{\emptyset}$, i.e. $(\tilde{Y}, \tilde{\sigma})$ is fuzzy semi-g-normal. \square

Recall that a fuzzy function $\tilde{f} : (\tilde{X}, \tilde{T}) \rightarrow (\tilde{Y}, \tilde{\sigma})$ is said to be fuzzy contra-semi-continuous if the inverse image of every fuzzy closed set in $(\tilde{Y}, \tilde{\sigma})$ is fuzzy semi-open in (\tilde{X}, \tilde{T}) .

Definition 8 : (i) A fuzzy space (\tilde{X}, \tilde{T}) is called weakly f s g-normal if disjoint f s g-closed sets can be separated by disjoint fuzzy closed sets.

(ii) A fuzzy function $\tilde{f} : (\tilde{X}, \tilde{T}) \rightarrow (\tilde{Y}, \tilde{\sigma})$ is said to be always f s g-closed if the image of each f s g-closed set in (\tilde{X}, \tilde{T}) is f s g-closed in $(\tilde{Y}, \tilde{\sigma})$.

Theorem 3.9 : If $\tilde{f} : (\tilde{X}, \tilde{T}) \rightarrow (\tilde{Y}, \tilde{\sigma})$ is an injective contra-fuzzy semi-continuous always f s g-closed function and $(\tilde{Y}, \tilde{\sigma})$ is weakly f s g-normal, then (\tilde{X}, \tilde{T}) is fuzzy semi-g-normal.

Proof: Suppose that $\tilde{A}_1, \tilde{A}_2 \leq \tilde{X}$ are f s g-closed and disjoint. Since \tilde{f} is always

f s g-closed and injective, $\tilde{f}(\tilde{A}_1), \tilde{f}(\tilde{A}_2) \leq \tilde{Y}$ are f s g-closed and disjoint. Since $(\tilde{Y}, \tilde{\sigma})$ is weakly f s g-normal, $\tilde{f}(\tilde{A}_1)$ and $\tilde{f}(\tilde{A}_2)$ can be separated by disjoint fuzzy closed sets $\tilde{B}_1, \tilde{B}_2 \leq \tilde{Y}$. Since \tilde{f} is contra-fuzzy semi-continuous, \tilde{A}_1 and \tilde{A}_2 can be separated by the disjoint fuzzy semi-open sets $\tilde{f}^{-1}(\tilde{B}_1)$ and $\tilde{f}^{-1}(\tilde{B}_2)$. Thus (\tilde{X}, \tilde{T}) is fuzzy semi-g-normal. \square

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